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Advanced Statistical Physics II – Problem Sheet 5

Problem 1 – **Functional derivative and Taylor expansion**

Given a functional space M and a function f, we define a functional as:

$$F : M \to \mathbb{R} \quad f(x) \equiv h \longmapsto F[f] \tag{1}$$

and using the definition of the delta function a functional derivative as:

$$\frac{\delta F[f]}{\delta f(t')} = \lim_{\epsilon \to 0} \frac{F[f(t) + \epsilon\delta(t - t')] - F[f(t)]}{\epsilon} \tag{2}$$

(a) (4P) Calculate $\frac{\delta F[f]}{\delta f(x)}$ and $\frac{\delta^2 F[f]}{\delta f(x)\delta f(y)}$ where:

$$F[f] = \int dxdy f(x)g(x-y)f(y) + \int dx f(x)h(x) \tag{3}$$

(b) (4P) The Taylor expansion for a functional $F[f]$ around a function $f_0(x)$ is:

$$F[f] = F[f_0] + \int dx_1 \frac{\delta F[f]}{\delta f(x_1)}\bigg|_{f=f_0} \big(f(x_1) - f_0(x_1)\big)$$
$$+ \frac{1}{2}\int dx_1 dx_2 \frac{\delta^2 F[f]}{\delta f(x_1)\delta f(x_2)}\bigg|_{f=f_0} \big(f(x_1) - f_0(x_1)\big)\big(f(x_2) - f_0(x_2)\big) + \dots$$

Expand the functional $F[f] = \int dx \big(f(x) + f(x)^2\big)$ around $f_0(x)$ to all non-vanishing orders.

(c) (2P) Assume the functional:

$$F[f] = \int_{t1}^{t2} dt\ L(f(t), \dot{f}(t), t) \tag{4}$$

If f extremizes F with fixed boundaries $\delta f(t_1) = \delta f(t_2) = 0$, show that the functional derivative fulfills the Euler-Lagrange equation.

Hint: Use a multidimensional Taylor-series.

Problem 2 – **First order linear ODE**

Consider the inhomogeneous ODE of the function $x = x(t)$

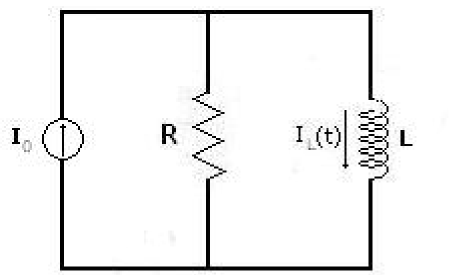
$$\dot{x} + p(t)x = q(t) \tag{5}$$

(a) (1P) First find a solution to the homogeneous ODE:

$$\dot{x} + p(t)x = 0 \tag{6}$$

(b) (2P) Then use it to solve the inhomogeneous ODE of eq. (5)
Hint: $\frac{d(ux)}{dt} = \dot{u}x + u\dot{x}$

(c) (3P) Calculate the current through an inductor $i_L(t)$ in an LR Circuit, with a sinusoidal power generator:



$$I_0 sin(\omega t) = \frac{V(t)}{R} + i_L(t) \tag{7}$$

Hint: $V(t) = L\frac{di_L}{dt}$

Problem 3 – **Piezoelectric Effect**

Consider the Hamiltonian:

$$\mathcal{H}(p,q,t) = \mathcal{H}_0(p,q) - E(t)P(q) - F(t)X(q) \tag{8}$$

where $E(t)$ is the electric field in x-direction, $P(q)$ is the polarization in x-direction, $F(t)$ external force in x-direction and $X(q)$ is the extension in x-direction. Assume an external force $F(t) = F_0\delta(t)$ and a constant electric field that it is switched on at $t = 0$, $E(t) = E_0\Theta(t)$.

(4P) Calculate $< \Delta P(t) >$ and $< \Delta X(t) >$ in terms of the correlation functions of P and X.