Freie Universität Berlin
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Statistical Physics and Thermodynamics (SS 2018)

Problem sheet 11

Hand in: Friday, July 13th during the lecture

http://www.physik.fu-berlin.de/en/einrichtungen/ag/ag-netz/lehre/

1 Joule-Thomson process (10 points)

Consider a pipe with thermally insulated walls. A porous plug divides the pipe into two parts. The porous plug in the pipe provides a constriction to the flow of the gas (alternatively, a valve which is only slightly opened may provide such a constriction). We consider a situation where gas flows from left to right. At the left side the gas has a pressure P_1 and a volume V_1. After the constriction the gas expands to a volume V_2 greater than the volume V_1. The presence of the constriction results in a constant pressure difference being maintained across this constriction. Thus the gas pressure P_1 to the left of the constriction is greater than the gas pressure P_2 to the right of the constriction. Let T_1 denote the temperature of the gas on the left-hand side of the constriction. In this exercise, we derive what the gas temperature T_2 on the right-hand side will be.

a) Write down the change of the internal energy, ΔU, and the net work W done by the floating gas. **(1 point)**

b) Use the fact that no heat is exchanged during the process and your result from a) to show that $H_2 = H_1$, where $H = U + PV$ is the enthalpy. **(1 point)**

c) Use the first law of thermodynamics to show that $\mathrm{d}H(S, P) = T\mathrm{d}S + V\mathrm{d}P$ **(1 point)**

d) In our case, where H is constant, $\mathrm{d}H(S, P) = 0$. Use this to show that

$$C_P\,\mathrm{d}T + V\,(1 - T\alpha)\,\mathrm{d}P = 0\,, \tag{1}$$

where C_P is the heat capacity at constant pressure and α is the coefficient of expansion. **(4 points)**

Hint: To obtain Eq. (1) *calculate the total differential* $\mathrm{d}S(T, P)$ *and find a suitable Maxwell relation for* $(\partial S/\partial p)_T$.

e) Use your previous result to calculate the so called Joule-Thomson coefficient $\mu_{\mathrm{JT}} := (\partial T/\partial P)_H$. What does this coefficient describe physically? **(1 point)**

f) Calculate μ_{JT} for the ideal gas. Argue what the temperature T_2 at the right-hand side of the constriction will be. **(1 point)**

g) Calculate μ_{JT} for the van der Waals gas. What does this result mean for the temperature T_2 at the right-hand side of the constriction? **(1 point)**

2 Two interacting dipoles (10 points)

Two identical ideal dipoles are located in a two-dimensional plane with a distance r between them. The dipoles are positioned at angles θ_1 and θ_2 relative to the connecting line. The interaction energy between the dipoles is given by

$$V(r, \theta_1, \theta_2) = -\frac{d^2}{4\pi\varepsilon r^3}\left[\frac{3}{2}\cos(\theta_1+\theta_2) + \frac{1}{2}\cos(\theta_1-\theta_2)\right],$$

with d being the dipole moment and ε being the dielectric constant of the medium between the dipoles.

a) Calculate the average interaction potential between two dipoles, $\bar{V}(r)$, from a Boltzmann-weighted integral over all possible orientations

$$\bar{V}(r) = -\ln \int_0^{2\pi} \frac{d\theta_1}{2\pi} \int_0^{2\pi} \frac{d\theta_2}{2\pi} \exp\left[-\frac{V(r,\theta_1,\theta_2)}{k_B T}\right]. \qquad \textbf{(4 points)}$$

Hint: The integral

$$\int_0^{2\pi} d\theta \exp\left[A\cos\theta\right] = 2\pi I_0(A),$$

can be written as

$$I_0(A) = \sum_{k=0}^{\infty} \frac{\frac{1}{4}\left(A^2\right)^k}{(k!)^2},$$

where $I_0(A)$ is the modified Bessel function of the first kind.

b) Expand $\bar{V}(r)$ to leading order for large distances r, which means $r^3 > d^2/(4\pi\varepsilon k_B T)$. How does the average interaction potential depend on the distance r? **(3 points)**

c) For large arguments A, the modified Bessel function scales like

$$I_0(A) \sim \frac{\exp A}{\sqrt{2\pi A}}.$$

Approximate $\bar{V}(r)$ to leading order for small distances r. How does the average interaction potential depend on the distance r? **(3 points)**