Name:

Matrikelnummer:

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Exam: Advanced Statistical Physics
Part II: Problems (75P)

1 Lenoir Cycle (25P)

Consider 1 mol of an ideal gas, which initially has a volume V_1 and temperature T_1 at pressure p_1. The gas undergoes the following cyclic process:
$1 \to 2$: isochoric (constant V) heating to T_2
$2 \to 3$: isentropic expansion to V_3
$3 \to 1$: isobaric cooling

a) Sketch the P-V and the T-S diagram for this cyclic process.

b) For each step calculate the performed work W and the heat transfer Q in terms of p_1, V_1 and V_3.

c) Calculate the efficiency η in terms of $\alpha = V_3/V_1$.

2 Adsorption (25P)

Consider an ideal gas (temperature T, chemical potential μ) in contact with a surface with N adsorption sites. Each adsorption site may be occupied by 0, 1 or 2 gas molecules. The energy of a vacant site is zero, the energy with one adsorbed molecule is $-\epsilon$ and the energy with two adsorbed molecules is $-(3/2)\,\epsilon$. ϵ can be positive or negative. There is no interaction between molecules at different adsorption sites.

a) Calculate the grand canonical partition function for a fixed number N of adsorption sites.

b) Use the grand canonical partition function to derive the mean number of adsorbed particles per site $\langle n \rangle$ and the mean internal energy per site $\langle u \rangle$ as a function of T, μ and ϵ.

c) For $T = 0$ sketch $\langle n \rangle$ for constant μ as a function of ϵ.

d) Calculate $\langle n \rangle$ for large temperatures. (No corrections in T are necessary.)

3 Spin 1/2 Fermions in an External Magnetic Field in 2 Dimensions (25P)

Consider an ideal gas of N spin 1/2 Fermions at zero temperature confined to an area A in two dimensions. The Fermions are in an external magnetic field H. The energy of a particle is $\epsilon = \frac{p^2}{2m} \pm \mu_B H$, where μ_B is the Bohr magneton.

a) Give an expression for the chemical potential μ_0 for vanishing magnetic field as a function of the particle density N/A.

b) Calculate the average particle energy as a function of μ_0 for weak external magnetic fields. Calculate corrections in H up to second order.

c) Calculate the susceptibility $\chi = \partial m/\partial H$ for weak external magnetic fields.