Prof. Dr. Roland Netz
Julius Schulz, Klaus Rinne

Second Chance Exam Solution: Advanced Statistical Physics
Part I: Questions (15P)

You have 15 minutes to work on part one of the exam. Please answer the questions on this sheet and write down your name and matrikelnumber. No notes or other tools are allowed in this part. You don't have to give explanations, short answers suffice. After 15 minutes, this sheet will be collected and part two will be handed out.

Good Luck!

1. How does the internal energy depend on the number of degrees of freedom of a given system?

$$U = fk_BT/2$$

2. If an ideal gas has f degrees of freedom per molecule, what is the value of the adiabatic coefficient γ?

$$\gamma = (f+2)/f$$

3. A system of volume V_1 and temperature T_1 changes adiabatically to V_2 and T_2. If γ is the adiabatic coefficient, what relation holds betweenV_1, V_2, T_1 and T_2?

$$V_1^{\gamma-1}T_1 = V_2^{\gamma-1}T_2$$

4. The thermal wavelength is $\lambda_t = \frac{h}{\sqrt{2\pi m}}(k_BT)^a$, where h is the Planck constant, m the particle mass, k_B the Boltzmann constant and T the temperature. Give the exponent a of the temperature dependence.

$$a = -1/2$$

5. Write down the Bose-Einstein distribution.

$$\langle n(E)\rangle = \frac{1}{e^{\beta(E-\mu)}-1}$$

6. $D(\epsilon) \propto \epsilon^a$ is the density of states at energy ϵ in 3D. Give the exponent a of the energy dependence for a non-relativistic Fermi gas.

$$a = 1/2$$

7. Give the Poisson distribution for λ being the expected number of occurences per interval and k the number of events occuring.

$$P_\lambda(k) = \frac{\lambda^k}{k!}\exp(-\lambda)$$

8. Let $p(x)$ be a probability distribution and $x \in (-\infty, \infty)$. What is the characteristic function of $p(x)$?

$$g(k) = \int_{-\infty}^{\infty} \mathrm{d}x\, p(x)e^{ikx}$$

9. Which thermodynamic potential is constant during the Joule-Thomson process?

$$H$$

10. What are the natural variables of the chemical potential?

$$p, T$$

11. Give the definition of the isothermal compressibility κ_T.

$$\kappa_T = -\frac{1}{V}\left(\frac{\partial V}{\partial p}\right)_T$$

12. The Hamiltonian of the 1D Ising model is: $H = -h\sum_{j=1}^{N} s_j - \epsilon\sum_{i,j} s_i\, s_j$. Write down the components of the transfer matrix T.

$$\begin{pmatrix} e^{\beta(2\,h+\epsilon)} & e^{-\beta\,\epsilon} \\ e^{-\beta\,\epsilon} & e^{\beta(-2\,h+\epsilon)} \end{pmatrix}$$

13. Let Z_1 be some canonical partition function of one particle. What is the grand canonical partition function if μ is the chemical potential and if there is no interaction between the indistinguishable particles?

$$Z_G = \sum_{N=0}^{\infty} \frac{Z_1^N}{N!}\, e^{\beta\,\mu\,N}$$

14. Let $\Gamma(E)$ be the phase space volume of a micro-canonical ensemble at energy E. What is the entropy of this system?

$$S = k_B \ln \Gamma(E)$$

15. Sketch the Lennard-Jones potential.