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# Second Chance Exam: Advanced Statistical Physics
# Part II: Problems (75P)

## 1 Three-Spin Interaction (20P)

Consider a one dimensional system of $N$ spins with the following Hamiltonian:

$$H = -J \sum_{i=1}^{N-2} S_i \, S_{i+1} \, S_{i+2},$$

with $S_i = \pm 1, i = 1...N$ being the spin states and $J = const$ being an interaction parameter. Assume open boundary conditions (**not** periodic!).

Calculate the canonical partition function and the Helmholtz free energy $F$. What is the thermodynamic limit of $F$?

*Hint: The transfer-matrix method is* ***not*** *necessary for this problem.*

*The definition of the cosinus hyperbolicus may be helpful:*

$$\cosh(x) = \frac{e^x + e^{-x}}{2}$$

## 2 Liquid-Gas Phase-Transition (25P)

We want to consider a substance with the enthalpy of the liquid phase

$$H_l(p, S) = 2 \, (a \, p \, S \, N)^{1/2}$$

and the enthalpy of the gas phase

$$H_g(p, S) = 3 \left( \frac{p \, S}{2} \right)^{2/3} (b \, N)^{1/3},$$

where $a > 0$ and $b > 0$ are constants, $S$ is the total entropy of the system, $p$ is the pressure and $N$ is the total number of particles.

a) Calculate the liquid-gas coexistence temperature $T_g$ as a function of pressure.

b) Calculate the densities of the liquid and the gas phase at the phase transition line.

c) Calculate the entropy change per volume $\Delta S / \Delta V$ at the phase transition line.

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## 3a Wien's Law in $n$ Dimensions (15P)

$u(\omega)$ is the energy density of black body radiation at angular frequency $\omega$ per volume and angular frequency, i.e. the spectral density of the internal energy density $U/V$.

Determine $u(\omega)$ in $n$ dimensions in the low temperature limit.

The volume of an $n$-dimensional sphere is $C_n R^n$, where $R$ is the radius. You do not have to determine $C_n$.

## 3b Cosmic Background Radiation (15P)

The universe is filled with a photon gas that corresponds to black body ratiation of temperature $T_{present} = 3\,\mathrm{K}$. In a simple view, this radiation arose from the isentropic expansion of a much hotter photon cloud, which was produced during the big bang.

If the volume of the universe, and thus the volume of the photon gas, increases isentropically by a factor of two starting from the present state, what will be the final temperature of the photon gas?

*Hint:* The Stefan-Boltzmann law can be useful.