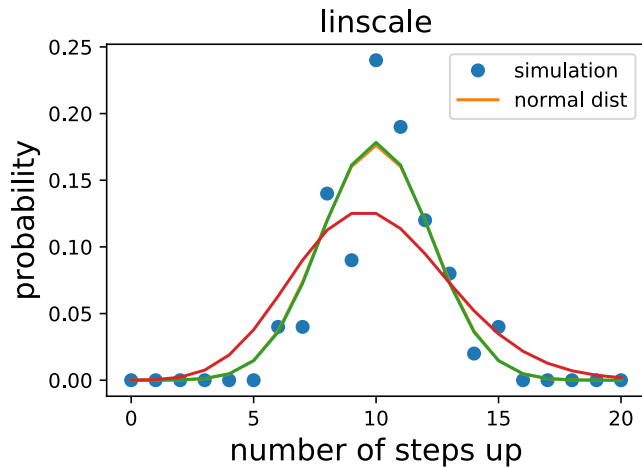
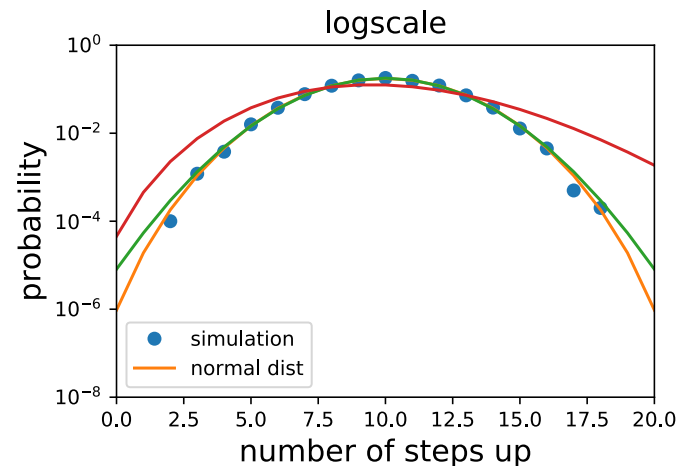
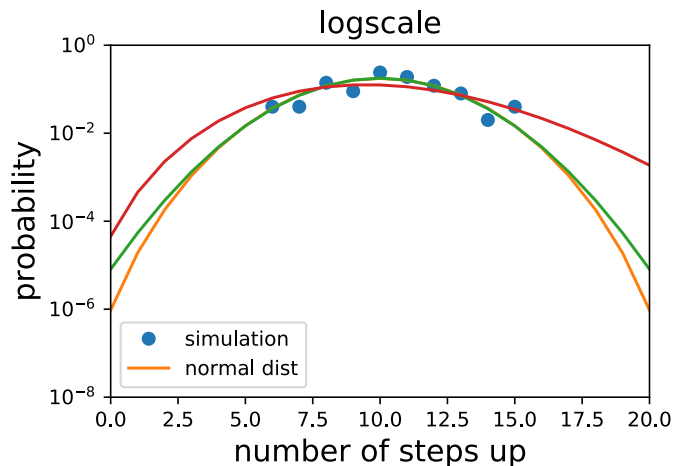
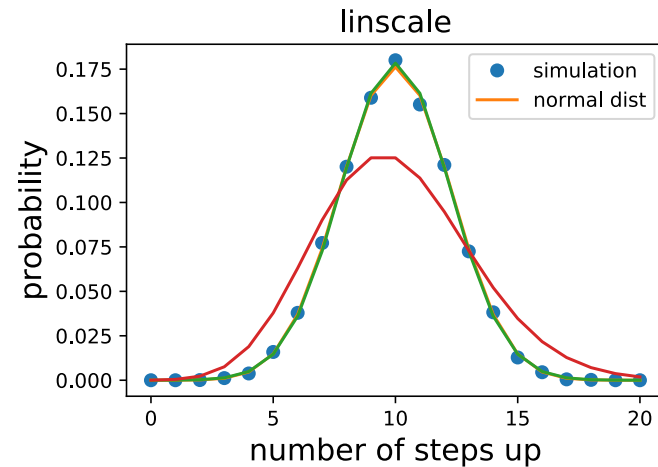


# Binomial distribution, comparison of simulation data with analytic distributions

random walk final position distribution for  $N=20$  steps  
100 random walks with up probability  $p=0.5$



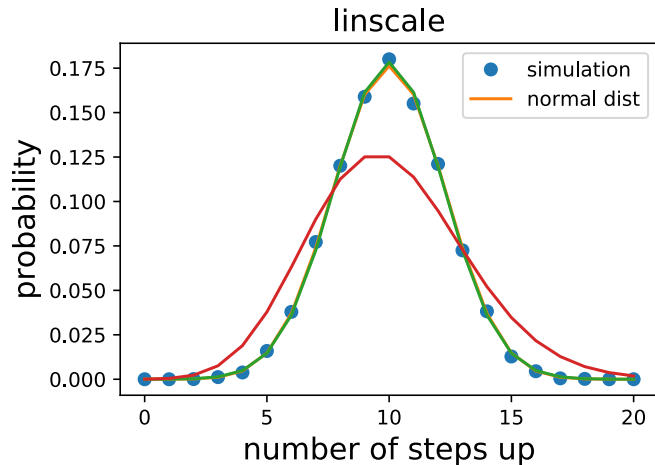
random walk final position distribution for  $N=20$  steps  
10000 random walks with up probability  $p=0.5$



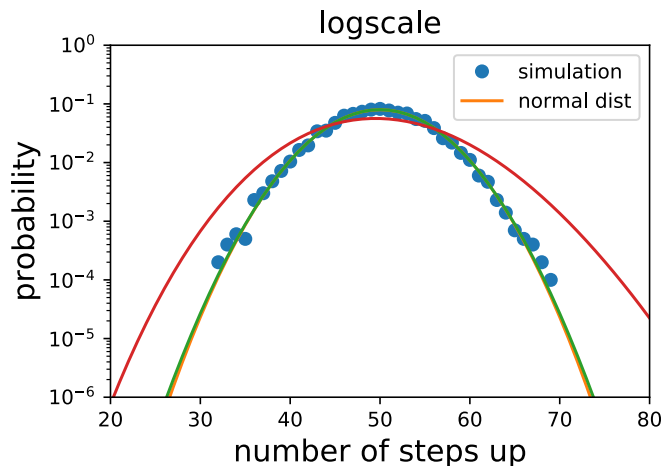
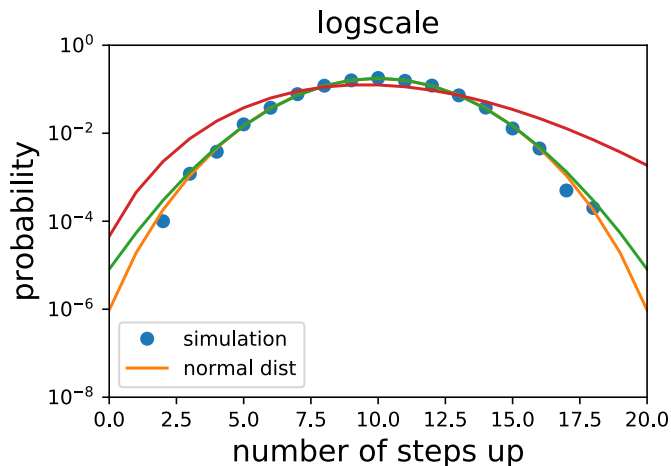
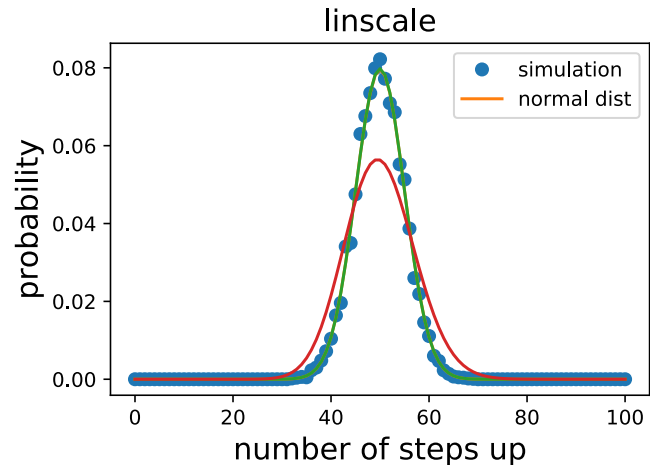
For  $N=20$  steps with probability  $p=0.5$  normal and binomial distributions are **very** close. With 100 random walks we see substantial numerical deviations from the normal distribution, with 10,000 random walks deviations are small. Poisson distribution does not describe the data.

# Binomial distribution, comparison of simulation data with analytic distributions

random walk final position distribution for N=20 steps  
10000 random walks with up probability p=0.5



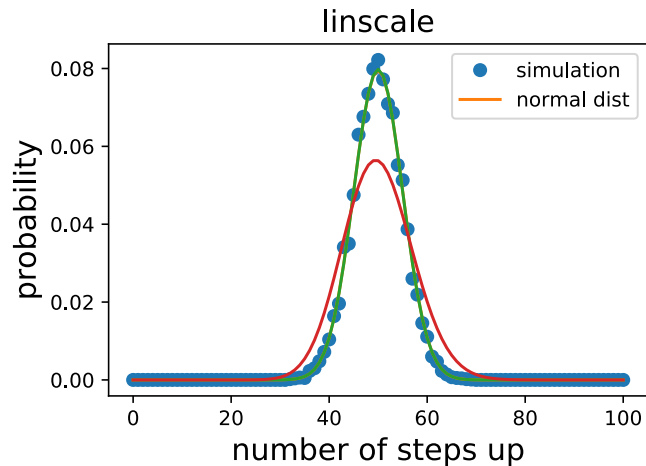
random walk final position distribution for N=100 steps  
10000 random walks with up probability p=0.5



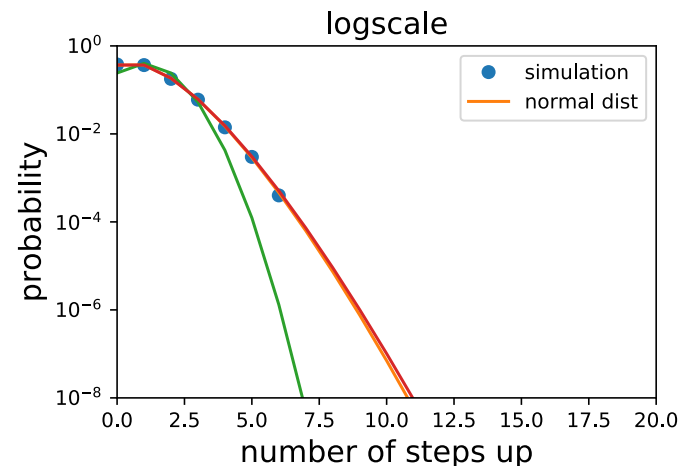
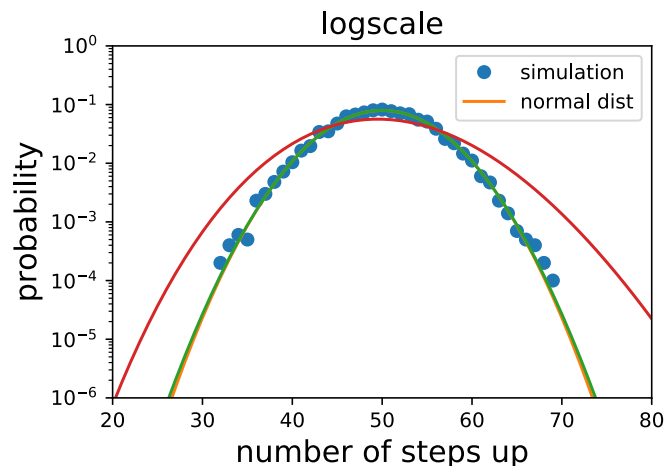
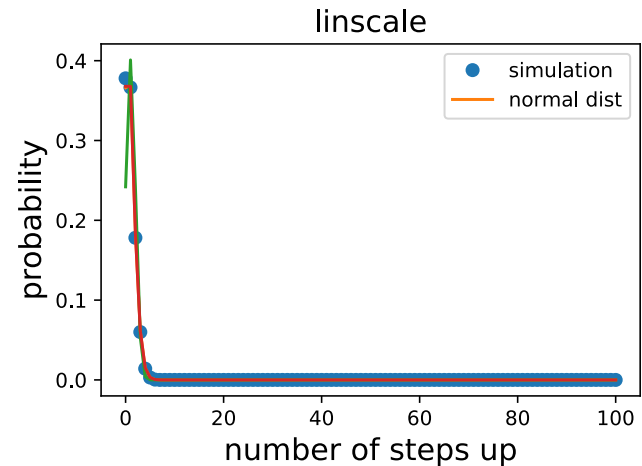
As we move from N=20 to N=100 steps at fixed probability p=0.5 the distribution becomes more sharply peaked. The mean deviation scales as  $N^{1/2}$ , while the relative mean deviation scales as  $N^{-1/2}$

# Binomial distribution, comparison of simulation data with analytic distributions

random walk final position distribution for  $N=100$  steps  
10000 random walks with up probability  $p=0.5$



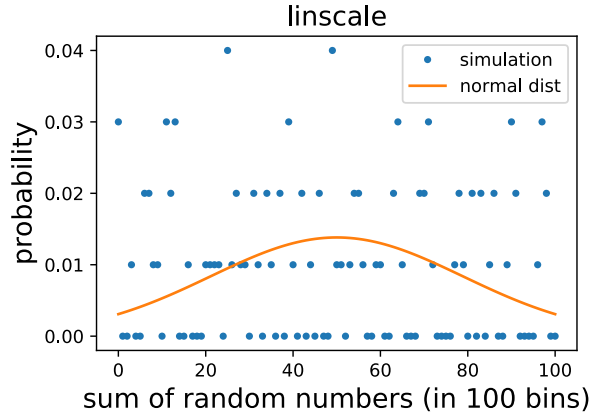
random walk final position distribution for  $N=100$  steps  
10000 random walks with up probability  $p=0.01$



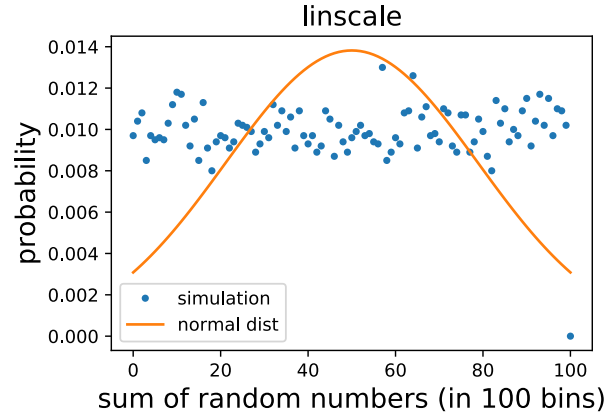
As we move from probability  $p=0.5$  to  $p=0.01$  for  $N=100$  steps  
the distribution shifts from „normal“ to „Poisson-like“.

# Central limit theorem: adding N uniform random numbers; first N=1

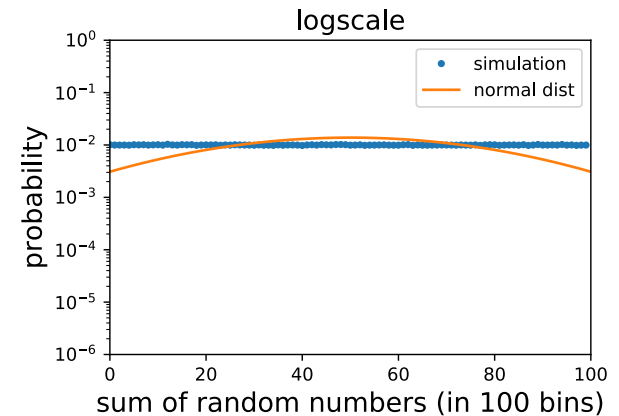
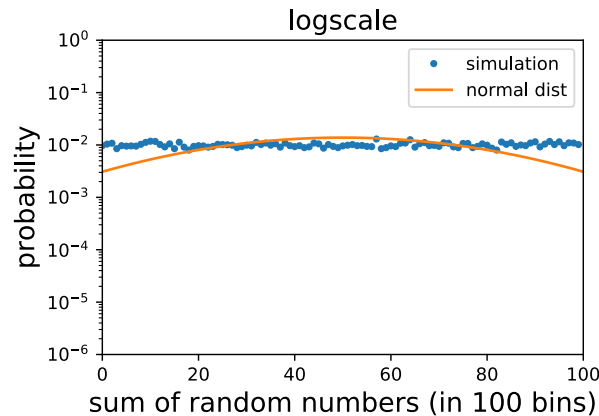
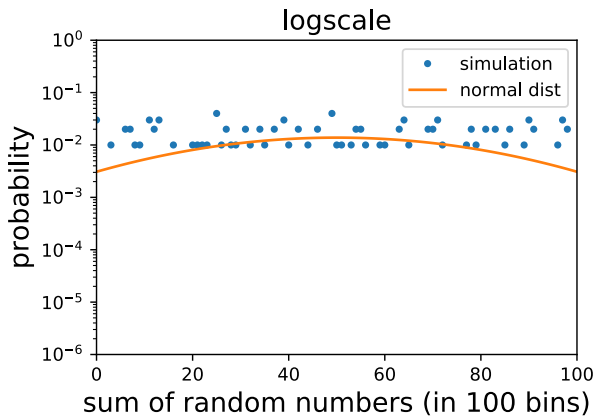
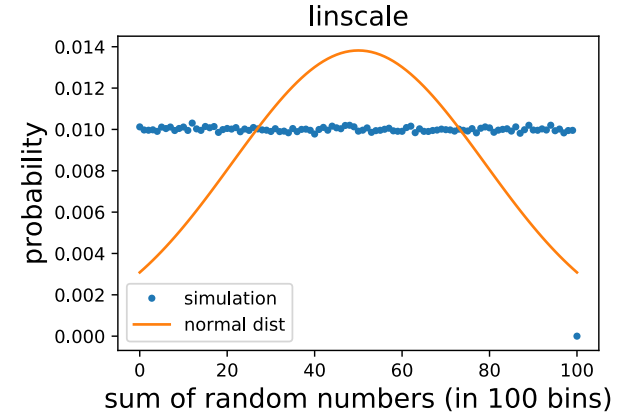
Distribution of sum of N=1 uniform random numbers in the interval 0 to 1 in  $M=1e+02$  realizations.



Distribution of sum of N=1 uniform random numbers in the interval 0 to 1 in  $M=1e+04$  realizations.



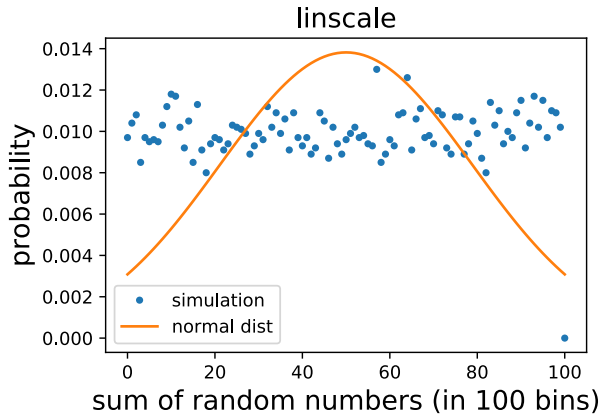
Distribution of sum of N=1 uniform random numbers in the interval 0 to 1 in  $M=1e+06$  realizations.



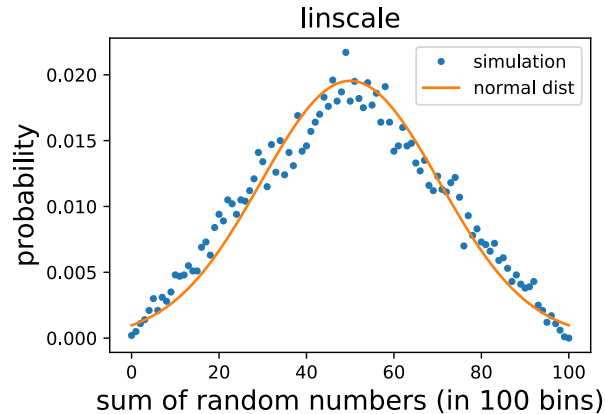
As we increase the number of realizations  $M$  from  $10^2$  over  $10^4$  to  $10^6$  the distribution becomes more uniform.

# Central limit theorem: adding N uniform random numbers; now N=1, 2, 3

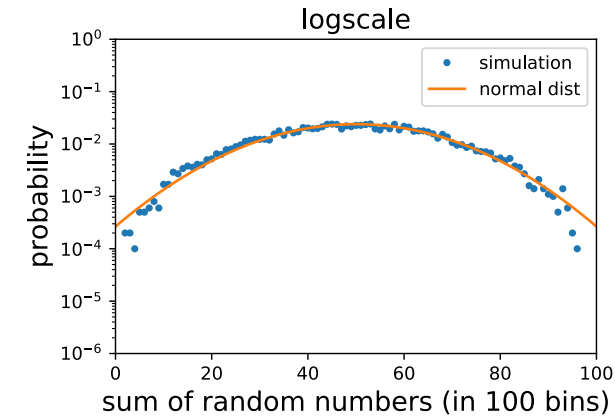
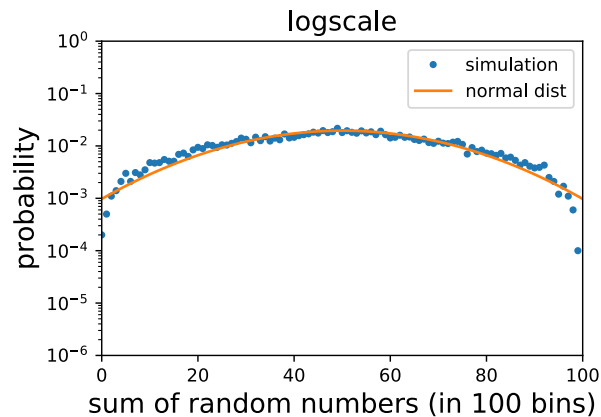
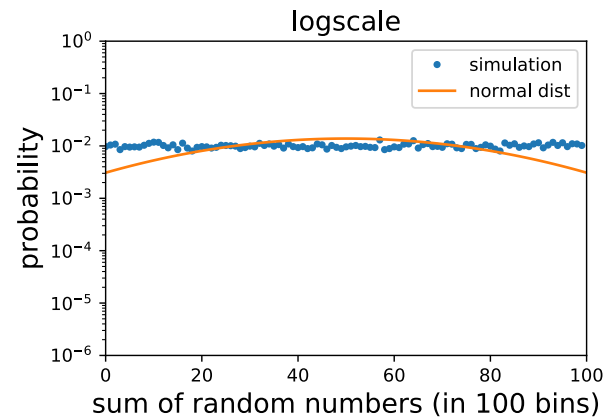
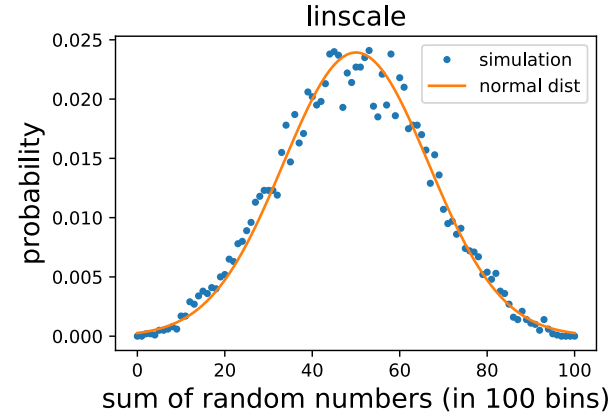
Distribution of sum of N=1 uniform random numbers in the interval 0 to 1 in  $M=1e+04$  realizations.



Distribution of sum of N=2 uniform random numbers in the interval 0 to 1 in  $M=1e+04$  realizations.



Distribution of sum of N=3 uniform random numbers in the interval 0 to 1 in  $M=1e+04$  realizations.



now we keep the number of realizations  $M$  fixed at  $10^4$

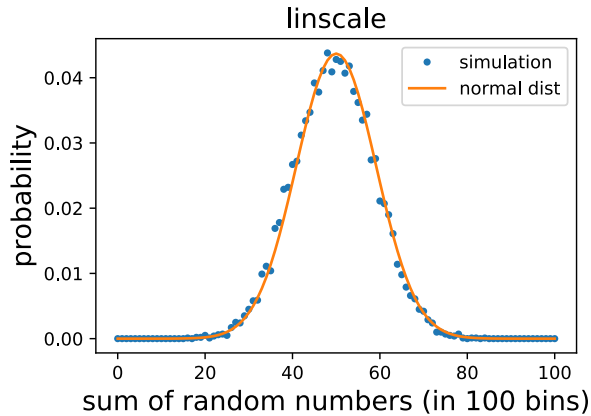
for N=2 the distribution is triangular (why?)

for N=3 the distribution already looks quite like a normal distribution.

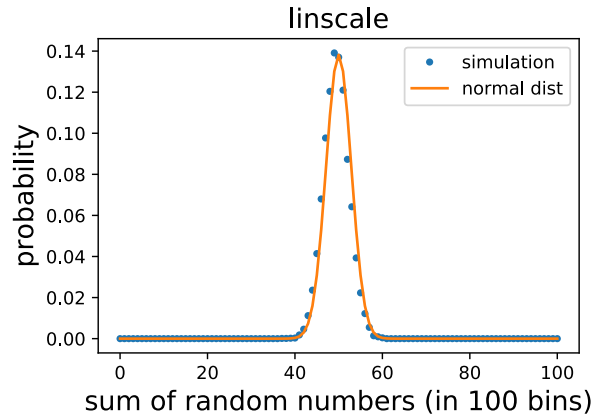
# Central limit theorem: adding N uniform random numbers; now N=10, 100, 1000

linear:

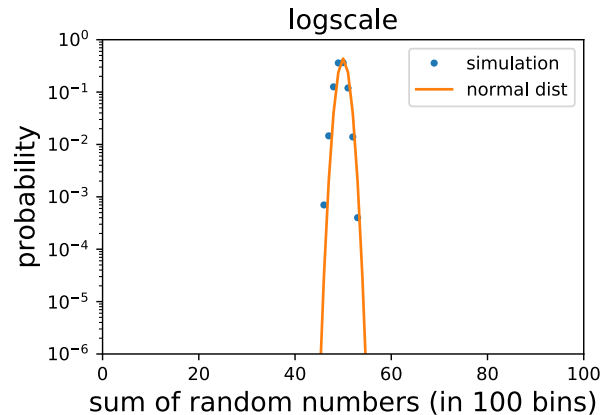
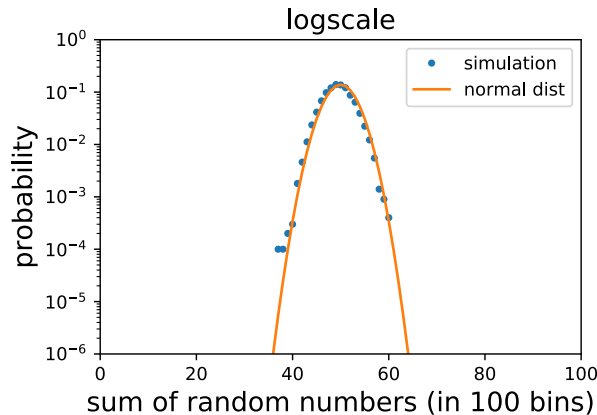
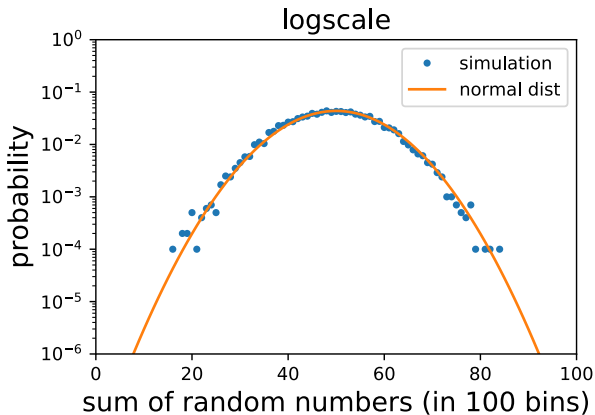
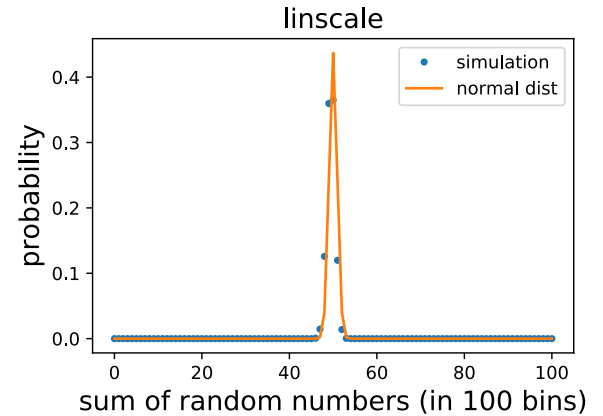
Distribution of sum of N=10 uniform random numbers in the interval 0 to 1 in M=1e+04 realizations.



Distribution of sum of N=100 uniform random numbers in the interval 0 to 1 in M=1e+04 realizations.



Distribution of sum of N=1000 uniform random numbers in the interval 0 to 1 in M=1e+04 realizations.



as N increases the distribution becomes sharper