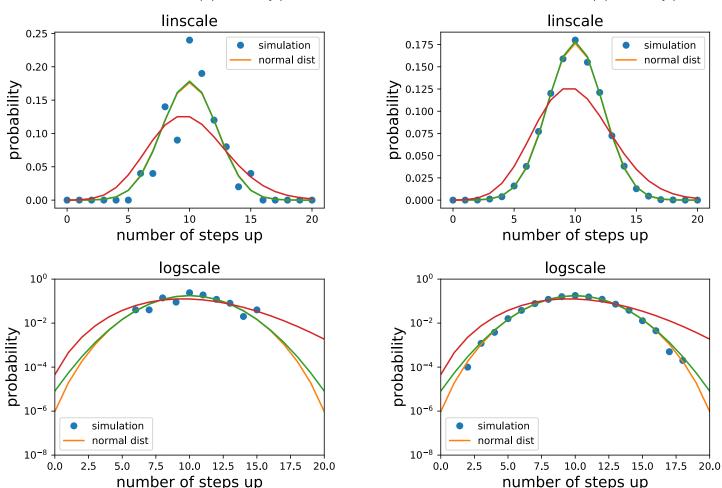
Binomial distribution, comparison of simulation data with analytic distributions

random walk final position distribution for N=20 steps

10000 random walks with up probability p=0.5



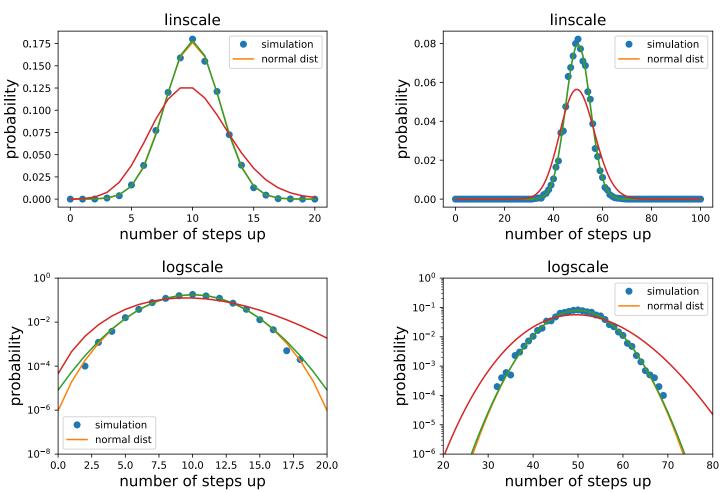
random walk final position distribution for N=20 steps 100 random walks with up probability p=0.5

For N=20 steps with probability p=0.5 normal and binomial distributions are **very** close. With 100 random walks we see substantial numerical deviations from the normal distribution, with 10,000 random walks deviations are small. Poisson distribution does not describe the data.

Binomial distribution, comparison of simulation data with analytic distributions

random walk final position distribution for N=20 steps

10000 random walks with up probability p=0.5



random walk final position distribution for N=100 steps 10000 random walks with up probability p=0.5

As we move from N=20 to N=100 steps at fixed probability p=0.5 the distribution becomes more sharply peaked. The mean deviation scales as N^{1/2}, while the relative mean deviation scales as N^{-1/2}

Binomial distribution, comparison of simulation data with analytic distributions

random walk final position distribution for N=100 steps 10000 random walks with up probability p=0.5

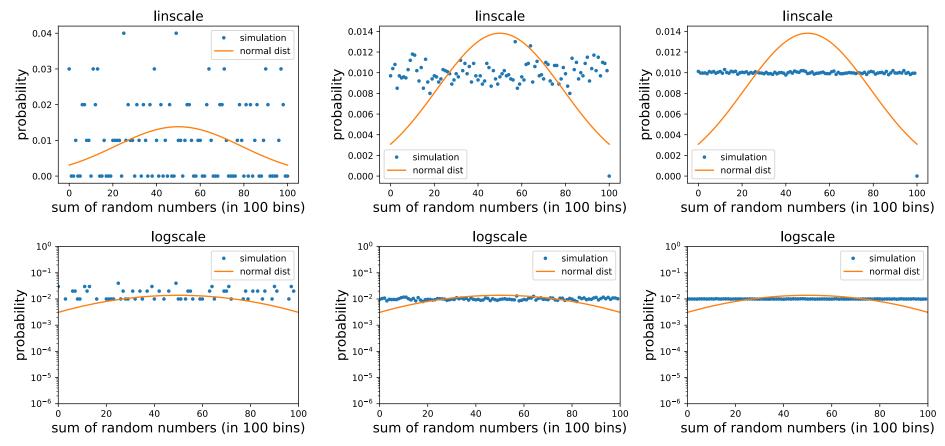
linscale linscale 0.4 0.08 simulation simulation • normal dist normal dist probability ... probability 0.3 0.1 0.02 0.00 0.0 20 40 60 80 100 20 40 60 80 100 0 0 number of steps up number of steps up logscale logscale 10⁰ 10^{0} simulation simulation • 10^{-1} normal dist normal dist 10-2 probability probability 10-2 10-3 10^{-4} 10^{-4} 10^{-6} 10-5 10^{-6} 10^{-8} 50 60 10.0 12.5 15.0 17.5 20.0 40 7.5 70 20 30 80 0.0 2.5 5.0 number of steps up number of steps up

As we move from probability p=0.5 to p=0.01 for N=100 steps the distribution shifts from "normal" to "Poisson-like".

random walk final position distribution for N=100 steps 10000 random walks with up probability p=0.01

Central limit theorem: adding N uniform random numbers; first N=1

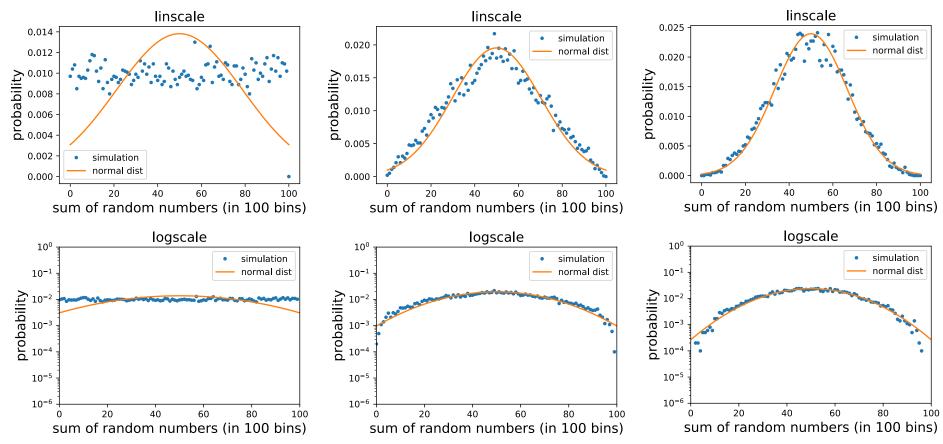
Distribution of sum of N=1 uniform random numbers in the interval 0 to 1 in M=1e+02 realizations. Distribution of sum of N=1 uniform random numbers in the interval 0 to 1 in M=1e+04 realizations. Distribution of sum of N=1 uniform random numbers in the interval 0 to 1 in M=1e+06 realizations.



As we increase the number of realizations M from 10^2 over 10^4 to 10^6 the distribution becomes more uniform.

Central limit theorem: adding N uniform random numbers; now N=1, 2, 3

Distribution of sum of N=1 uniform random numbers in the interval 0 to 1 in M=1e+04 realizations. Distribution of sum of N=2 uniform random numbers in the interval 0 to 1 in M=1e+04 realizations. Distribution of sum of N=3 uniform random numbers in the interval 0 to 1 in M=1e+04 realizations.



now we keep the number of realizations M fixed at 10⁴

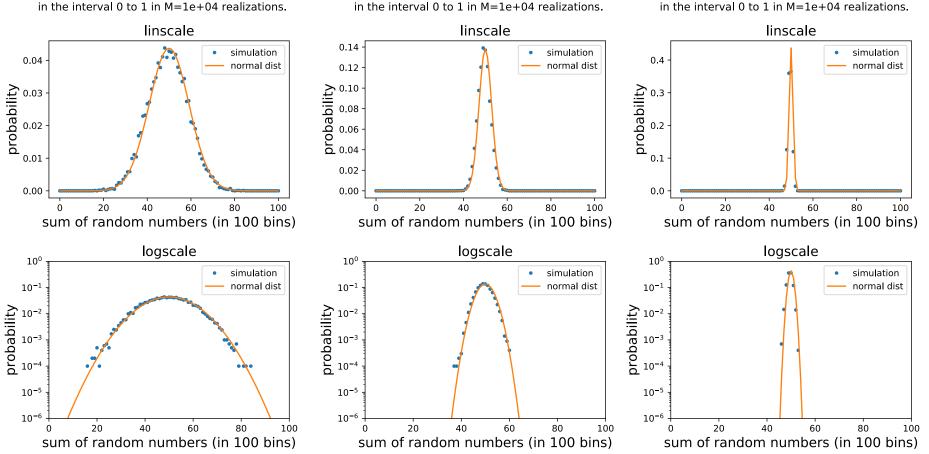
for N=2 the distribution is triangular (why?)

for N=3 the distribution already looks quite like a normal distribution.

Central limit theorem: adding N uniform random numbers; now N=10, 100, 1000

linear:

Distribution of sum of N=10 uniform random numbers



Distribution of sum of N=100 uniform random numbers Distribution of sum of N=1000 uniform random numbers in the interval 0 to 1 in M=1e+04 realizations. in the interval 0 to 1 in M=1e+04 realizations.

as N increases the distribution becomes sharper